Secondary math 3H
Final Exam Review

1. $$\frac{3x-17}{(x+5)(x-3)} = \frac{A}{x+5} + \frac{B}{x-3}$$

$$3x-17 = A(x-3) + B(x+5)$$

let $x = 3$

$3(3)-17 = A(3-3) + B(3+5)$

$9-17 = 8B$

$\frac{-8}{8} = \frac{8B}{8}$

$-1 = B$

let $x = -5$

$3(-5)-17 = A(-5-3) + B(-5+5)$

$\frac{-32}{-8} = \frac{-8A}{-8}$

$4 = A$

$$\boxed{\frac{4}{x+5} + \frac{-1}{x-3}}$$

2. 10 units to the right, 2 units to the downward

3. factor of 2, across the x-axis, 9, upward.

4. $a \cdot b = (4)(3) + (1)(-2) = 12 - 2 = 10$

5. $\sqrt{3}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = \sqrt{3}\left(\frac{\sqrt{3}}{2} + i\left(\frac{1}{2}\right)\right) = \boxed{\frac{3}{2} + \frac{\sqrt{3}}{2}i}$

6. $(5\cos 70° + i\sin 70°)^4 = 5^4(\cos(4\cdot 70°) + i\sin(4\cdot 70°))$

$= 625(\cos 280° + i\sin 280°)$

$= \boxed{108.53 - 615.50i}$

7. $y = 5\cos\frac{1}{2}x$ period $= \dfrac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2 = \boxed{4\pi}$

8. graph $y_1 = |x-1|$
$y_2 = 14x+4$ Find x-value of the intersection.

$\boxed{x = -\frac{1}{5}}$

9. $\log_5\left(\dfrac{x^6y^5}{7}\right) = (\log_5 x^6 + \log_5 y^5) - \log_5 7$

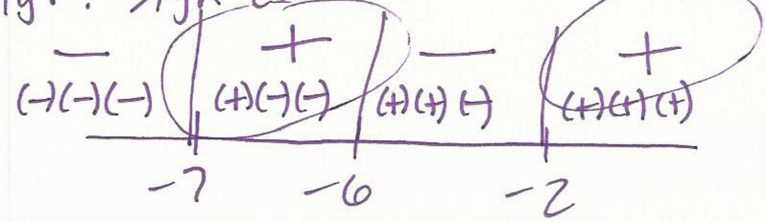
$= \boxed{(6\log_5 x + 5\log_5 y) - \log_5 7}$

10. $(x+7)(x+6)(x+2) > 0$

Can solve graphically or Algebriacally.

Graph: $(-7,-6) \cup (2,\infty)$

Alg.: Sign chart:



11. $(-2\sqrt{3}, 2\sqrt{3})$

$r = \sqrt{(-2\sqrt{3})^2 + (2\sqrt{3})^2}$

$= \sqrt{12 + 12} = \sqrt{24} = 2\sqrt{6}$

$\theta = \tan^{-1}\left(\frac{2\sqrt{3}}{-2\sqrt{3}}\right) = \tan^{-1}(-1) = -45°$ (in 4th Quad. need angle in 2nd Quad. add 180°)

$\theta = -45° + 180° = 135°$

$\boxed{(2\sqrt{6}, 135°)\ (-2\sqrt{6}, 315°)}$

12. $\frac{2x^3 - 3x^2 + 4x - 10}{x+1}$

$$\begin{array}{r|rrrr} -1 & 2 & +3 & 4 & -10 \\ & \downarrow & -2 & -1 & -3 \\ \hline & 2 & 1 & 3 & -13 \end{array} \rightarrow \boxed{(2x^2 + x + 3) + \frac{-13}{x+1}}$$

13. use the points from the table to match the graph. (Graph A)

14. $a_1 = -5$, $a_n = a_{n-1} + 8$

$a_1 = -5$

$a_2 = -5 + 8 = 3$

$a_3 = 3 + 8 = 11$

$a_4 = 11 + 8 = 19$

$a_5 = 19 + 8 = 27$

$a_6 = 27 + 8 = 35$

15.

$x = 2\cos t, \quad y = 2\sin t$

$x^2 + y^2 = 4\cos^2 t + 4\sin^2 t$

$x^2 + y^2 = 4(\underbrace{\cos^2 t + \sin^2 t}_{1})$

$\boxed{x^2 + y^2 = 4}$

16. $\cos\theta = \frac{-\sqrt{2}}{2}, \quad \tan\theta > 0$

$\theta = \frac{3\pi}{4}$ or $\frac{5\pi}{4}$, $\tan\theta > 0$ so, $\boxed{\theta = \frac{5\pi}{4}}$

17. $f(x) = x^3 - 2x^2 - 6x + 12$

1) use calc. to find one rational zero.

$x = 2$

2) use rational zero w/ syn. division to find quad.

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -6 & 12 \\ & & 2 & 0 & -12 \\ \hline & 1 & 0 & -6 & 0 \end{array}$$

$x^2 - 6 = 0$

$x^2 = 6$

$x = \pm\sqrt{6}$

zeros: 2; rational

$\sqrt{6}$; irrational

$-\sqrt{6}$; irrational

18. Quad. shifted to the left 4 units open downward.

$y = x^2 \xrightarrow{\text{left } 4} y = (x+4)^2 \xrightarrow{\text{down}} \boxed{y = -(x+4)^2}$

19. $\sin^2 x + \sin x = 0$

$\sin x(\sin x + 1) = 0$

$\sin x = 0 \quad , \quad \sin x + 1 = 0$

$x = 0, \pi \quad , \quad \sin x = -1$

$x = 3\pi/2$

20. $\cos^{-1}(\cos x) = 1.8$

$\boxed{x = 1.8}$

21. $f(x) = (x+3)^2(x-1)$

Zeros: $x = -3$, mult. 2
$x = 1$, mult. 1

22. $\log_6 x \rightarrow \boxed{\dfrac{\log x}{\log 6}}$

23. $z_1 = 3(\cos(-75^\circ) + i\sin(-75^\circ))$

$z_2 = 81(\cos(100^\circ) + i\sin(100^\circ))$

$z_1 \cdot z_2 = 3 \cdot 81(\cos(-75^\circ + 100^\circ) + i\sin(-75^\circ + 100^\circ))$

$= \boxed{243(\cos 25^\circ + i\sin 25^\circ)}$

24. $f(x) = 2x^3 + 7x^2 + 8x - 8$

$\dfrac{\text{factors of } 8}{\text{factors of } 2} \rightarrow \dfrac{\pm1, \pm2, \pm4, \pm8}{+1, \pm2} \rightarrow \boxed{\pm1, \pm2, \pm4, \pm8, \pm 1/2}$

25.

$$3^{-x} = \frac{1}{27}$$

$$3^{-x} = \frac{1}{3^3}$$

$$3^{-x} = 3^{-3}$$

$$\boxed{x = 3}$$

or

$$3^{-x} = \frac{1}{27}$$

$$\log_3 3^{-x} = \log_3\left(\frac{1}{27}\right)$$

$$-x\,\cancel{\log_3 3} = \log_3\left(\frac{1}{27}\right)$$

$$-x = \log_3\left(\frac{1}{27}\right) = \frac{\log\left(\frac{1}{27}\right)}{\log(3)}$$

$$\boxed{x = 3}$$

26. $x+1 = \frac{2}{x}$

$$x\left(x+1 = \frac{2}{x}\right)$$

$$x^2 + x = 2$$

$$x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0$$

$$\boxed{x = -2,\ x = 1}$$

27. $9, 18, 36, 72 \ldots$

$r = 2$

$a_1 = 9$

$a_n = a_1 \cdot r^{n-1}$

$$\boxed{a_n = 9 \cdot 2^{n-1}}$$

product rule → if (−) then quotient rule

28. $5\log_4(3x-3) + 3\log_4(5x+6)$

$$= \boxed{\log_4 (3x-3)^5 (5x+6)^3}$$

29. $r = \cos\theta$ → multiply the equation by r

$$r \cdot r = r \cdot \cos\theta$$

$$r^2 = r\cos\theta$$

$$\boxed{x^2 + y^2 = x}$$

30. $\csc 60^\circ = \dfrac{1}{\sin 60^\circ}$ (use unit circle)

$$\frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$$

31. $5-5i$ $\quad r = \sqrt{(5)^2+(-5)^2} = \sqrt{50} = 5\sqrt{2}$

$\theta = \tan^{-1}\left(\frac{-5}{5}\right) = -45^\circ$ or $-\pi/4 = \frac{7\pi}{4}$

Check what quadrant $5-5i$ is in (4^{th})

$\theta = 7\pi/4$ in 4^{th} so, ok to use this angle.

$$\boxed{5\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)}$$

32. $f(x) = 2x^3+3x^2+4x+18$; $k=-2$

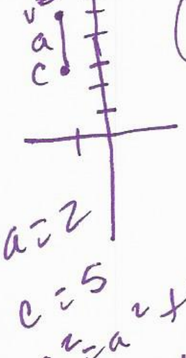
$f(-2) = 2(-2)^3+3(-2)^2+4(-2)+18 = \boxed{6}$

33. $(y+2)^2 = -8(x-1)$ → opens to the left

$\boxed{v(1,-2),\ f(-1,-2),\ \text{focal width } 8,\ \text{directrix } x=3}$

34. $c(-1,3)$, $f(-1,8)$, $v(-1,5)$

(vertical)



$$\boxed{\frac{(y-3)^2}{4} - \frac{(x+1)^2}{21} = 1}$$

$a=2$

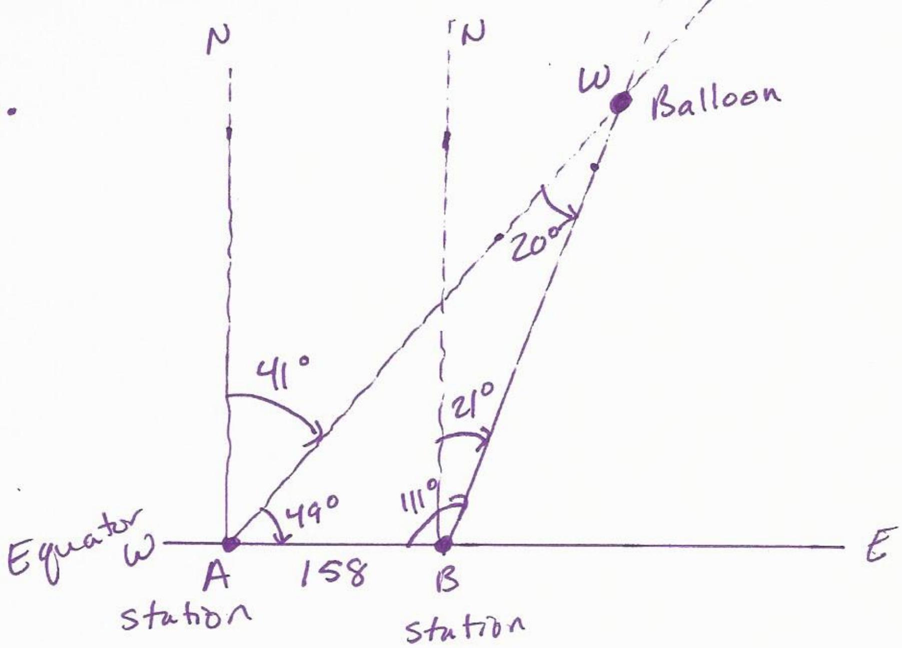
$c=5$

$c^2 = a^2+b^2$

$25 = 4+b^2$

$21 = b^2$

35.

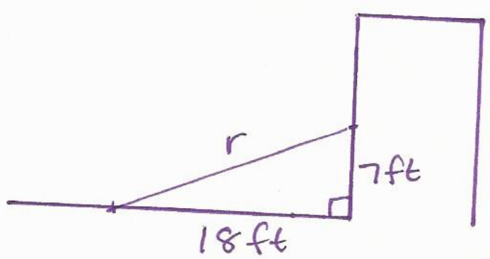


Find AW: $\frac{\sin 20°}{158} = \frac{\sin 111°}{AW}$

$AW = \frac{158 \sin(111°)}{\sin(20°)} = 431.27$

≈ 431 miles

36.



$18^2 + 7^2 = r^2$

$324 + 49 = r^2$

$\sqrt{373} = \sqrt{r^2}$

$\boxed{19.3 = r}$